JEE-Main-24-02-2021-Shift-1

PHYSICS

Question: Match the following for the value of 'α' for following process represented by PV^{α} = constant?

a) constant pressure	p) $\alpha = 0$
b) constant volume	q) $\alpha = 1$
c) constant temperature	r) $\alpha \to \infty$
d) No heat exchange	s) $\alpha = \gamma$

Options:

(a) a → q, b → s, c → p, d → r
(b) a → p, b → r, c → q, d → s
(c) a → s, b → p, c → r, d → q
(d) a → p, b → r, c → q, d → s

Answer: (b)

Solution:

PV^{α} = constant

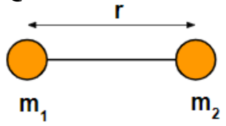
For adiabatic process, $PV^{\gamma} \Rightarrow a = \gamma (\text{for d})$

For isothermal process, PV = constant $\Rightarrow \alpha = 1$ (for c)

For isobaric process, P = constant $\Rightarrow \alpha = 0$

From these facts, options (B) follows.

Question: In a double star system. Find w.



Options:

(a) $w = \sqrt{\frac{G(m_1 + m_2)}{r^3}}$

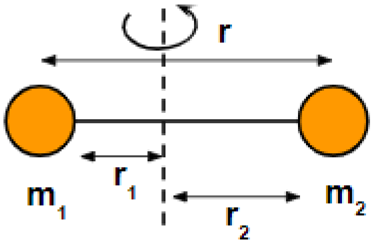
(b) $w = \sqrt{\frac{Gm_1 m_2}{(m_1 + m_2) r^3}}$

(c) $w = \sqrt{\frac{Gm_1}{r^3}}$

(d) $w = \sqrt{\frac{Gm_2}{r^3}}$

Answer: (a)

Solution:



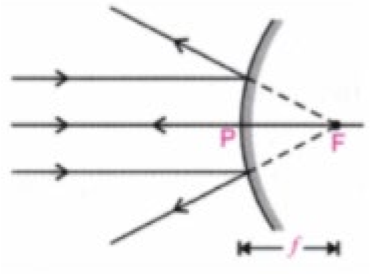
$$\frac{Gm_1m_2}{r^2} = m_1\omega^2 r_1$$

$$\frac{Gm_1m_2}{r^2} = m_1\omega^2 \frac{m_2 r}{m_1+m_2}$$

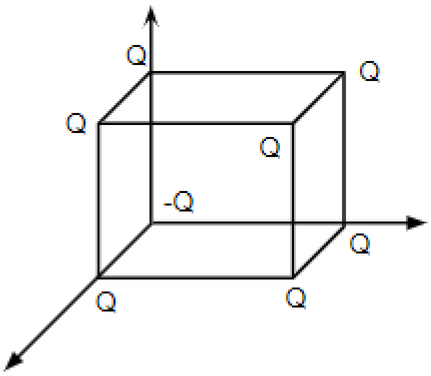
$$\omega^2 = \frac{G(m_1+m_2)}{r^3}$$

$$\omega = \sqrt{\frac{G(m_1+m_2)}{r^3}}$$

Question: What is the sign of focal length of convex mirror?
Options:
(a) +ve
(b) -ve
(c) Can be +ve or -ve
(d) None of these
Answer: (a)
Solution:
The focal length of a convex mirror is positive
(as per cartesian sign conventions)



Question: Find field at center of cube of side a.



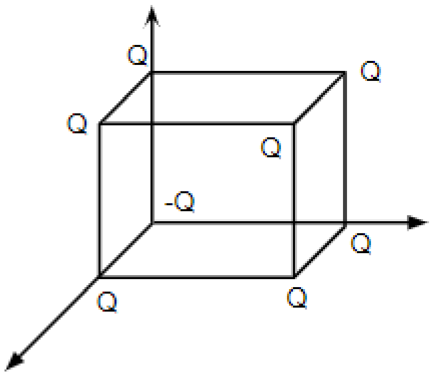
Options:

(a) $\frac{4kQ}{3a^2}$
(b) $\frac{8kQ}{3a^2}$
(c) $\frac{16kQ}{3a^2}$
(d) None of these
Answer: (b)
Solution:
If r is the distance from any vertex to the centre of the cube, then;

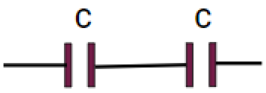
$$\left(v = \frac{a\sqrt{3}}{2}\right)$$

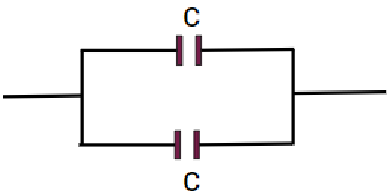
$$E = 2 \times \frac{KQ}{a^3(3)} \times 4$$

$$= \frac{8}{3}\frac{KQ}{a^2}$$



Question: Two equal capacitors are as shown in the two figure I and II. The ratio of the equivalent capacitance in the two diagrams is:-





Options:
(a) $\frac{C_I}{C_{II}} = \frac{1}{2}$
(b) $\frac{C_I}{C_{II}} = 1$
(c) $\frac{C_I}{C_{II}} = \frac{1}{4}$
(d) None of these

Answer: (c)
Solution:

When in series, equivalent capacitance: $C_I = \frac{C}{2}$

When in parallel, equivalent capacitance: $C_{II} = 2C$

$\frac{C_I}{C_{II}} = \frac{1}{4}$

Question: Compare the magnitudes of moment of Inertia of (masses & Radius are equal)
(a) Ring About Diameter (I_a)
(b) Disc About $\perp$ axis passing through centre (I_b)
(c) SOLID cylinder about (Axis) (I_C)
(d) SOLID sphere (I_D)
Options:
(a) $I_a = I_b = I_C > I_D$
(b) $I_a = I_b > I_C > I_D$
(c) $I_a < I_b < I_C < I_D$
(d) None of these
Answer: (a)
Solution:

$I_a = \frac{1}{2}MR^2$

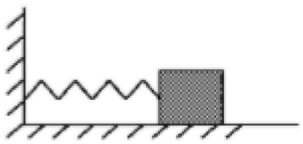
$I_b = \frac{1}{2}MR^2$

$I_c = \frac{1}{2}MR^2$

$I_d = \frac{2}{5}MR^2$

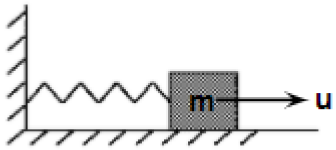
$I_a = I_b = I_c > I_d$

Question: A horizontal spring block system (mass m_2 spring constant k) is oscillating on a smooth surface with amplitude A. If at the mean position an identical block is kept on it so that they move together after what is the new amplitude?



Options:
(a) A
(b) $\frac{A}{2}$
(c) $\frac{A}{\sqrt{2}}$

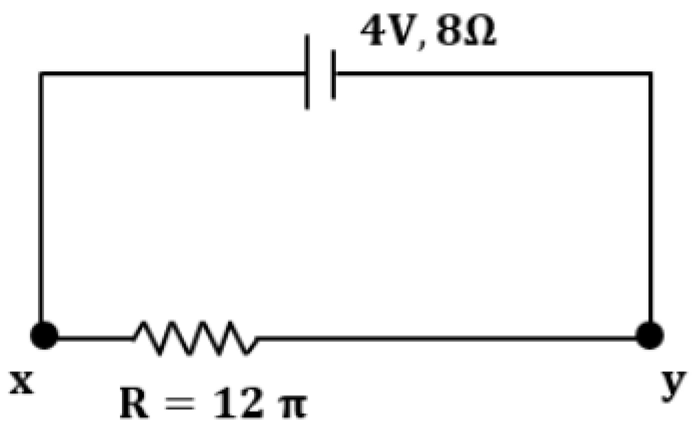
(d) $\frac{A}{2\sqrt{2}}$
Answer: (c)
Solution:



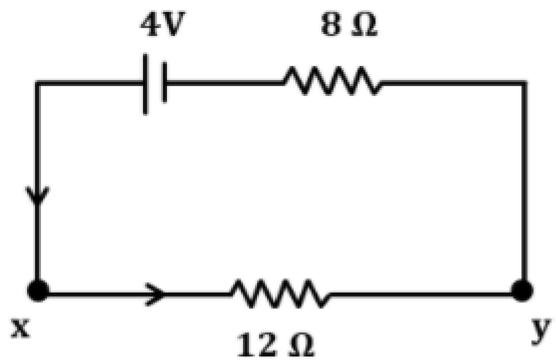
$$\frac{1}{2}KA^2 = \frac{1}{mu^2}$$
$$\left(u^2 = \frac{K}{m}A^2\right)$$
$$mu = (2m)v\left(v = \frac{u}{2}\right)$$
$$\frac{1}{2}(2m)v^2 = \frac{1}{2}KA'^2$$
$$\frac{1}{2}(2M)\frac{u^2}{4} = \frac{1}{2}KA'^2$$
$$A' = \frac{A}{\sqrt{2}}$$

Question: How does the energy of photon change if the corresponding wavelength increases?
Options:
(a) Increases
(b) Decreases
(c) May increase or decrease
(d) Doesn’t depend on wavelength
Answer: (b)
Solution:
$E \propto v$
But $v \propto \frac{1}{\lambda}$
$\therefore E \propto \frac{1}{\lambda}$
As wavelength increases, energy decreases.

Question: In the shown circuit the following the battery is of EMF 4 and internal resistance 8 Ω. Find the potential difference between points x and y is shown in the circuit.



Options:
(a) $V_x - V_y = 2.4V$
(b) $V_y - V_x = -2V$
(c) $V_x - V_y = 4V$
(d) $V_y - V_x = -4V$
Answer: (a)
Solution:



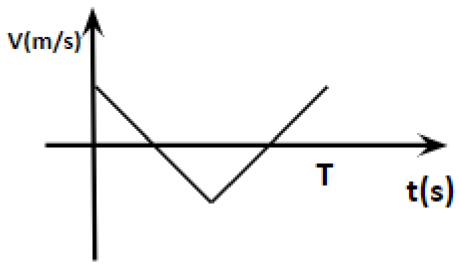
$i = \frac{4}{20} = \frac{2}{10}$

$V_x - V_y = i(12)$

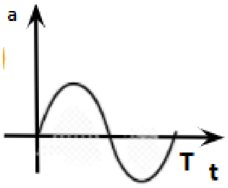
$= \frac{2}{10} \times 12$

$= 2.4V$

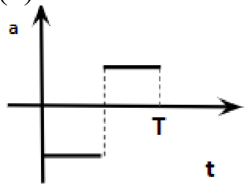
Question: Choose the correct a v/s t graph



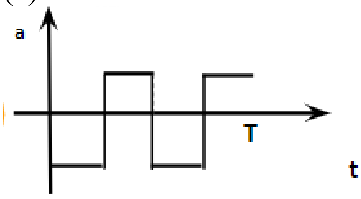
Options:
(a)



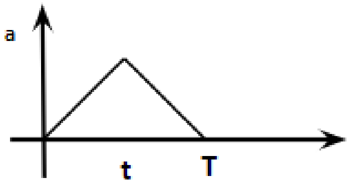
(b)



(c)



(d)



Answer: (b)
Solution:
Slope of velocity - time graph gives acceleration.
For the first part, slope is negative and constant. Means acceleration has a steady negative value.
For the second part, slope is positive and constant. Acceleration has a steady positive value

Question: If μ_r & ε_r represents relative permeability of Medium & Relative permittivity of medium & speed of light in medium is $\frac{c}{\mu}$, where μ (refractive Index) will be -

Options:
(a) $\varepsilon_r\mu_r$
(b) $\sqrt{\in_r \mu_r}$
(c) $\frac{1}{\sqrt{\in_r \mu_r}}$
(d) None of these
Answer: (b)
Solution:

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$v = \frac{c}{\mu} = \frac{1}{\sqrt{\mu_m\varepsilon_m}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$$

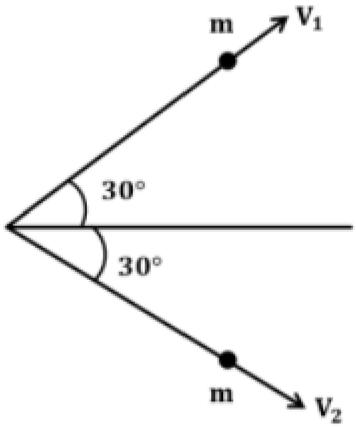
$$= \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \times \frac{1}{\sqrt{\mu_r \varepsilon_r}}$$

$$= \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

$$\Rightarrow \mu = \sqrt{\mu_r \varepsilon_r}$$

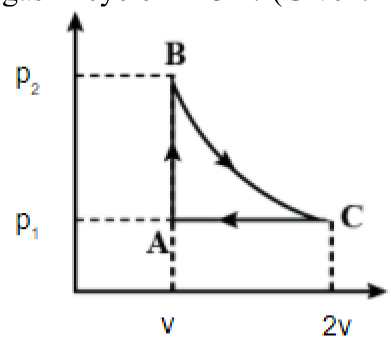
Question: A body cools from 100ºC to 90ºC in 20 mins. Find the time to cool from 110ºC to 100ºC.
Options:
(a) Equal to 20 mins
(b) Less than 20 mins
(c) More than 20 mins
(d) 30 min
Answer: (b)
Solution:
By Newton's law of cooling: $\frac{dQ}{dt} \propto (T - T_0)$
$T - T_0$ is the temperature difference between the body and the surroundings.
If this difference is more, then rate of cooling is higher, or cooling is faster.
So, to cool from $110°C$ to $100°C$ takes less time compared to the time taken to cool from $100°C$ to $90°C$

Question: An object is moving with speed v and collides with an identical mass at rest. After collision, they both move at an angle of 30º with original direction. Find ratio of final speed of mass to the initial speed?
Options:
(a) $\frac{1}{2}$
(b) $\frac{2}{1}$
(c) $\frac{\sqrt{3}}{1}$
(d) $\frac{1}{\sqrt{3}}$
Answer: (d)
Solution:

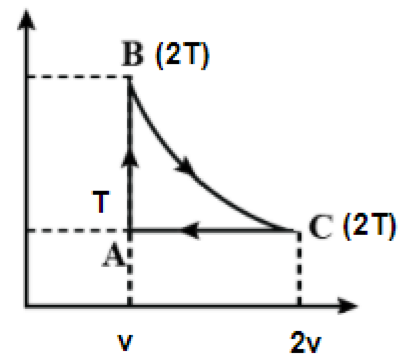


$u = mV\cos 30° + mV\cos 30°$
$u = 2V\cos 30°$
$\frac{V}{u} = \frac{1}{2\cos 30°} = \frac{1}{\sqrt{3}}$

Question: In a cyclic process ABCA, the process BC is isothermal. Find the work done by gas in cycle ABCA? (Given: $P_1 = P$)



Options:
(a) 2ln2 PV
(b) (2ln2 - 1) PV
(c) PV ln2
(d) None of these
Answer: (b)
Solution:
$W = W_{AB} + W_{BC} + W_{CA}$
$W = 0 + nR(2T)\ln\frac{2v}{v} + nR(T - 2T)$
$W = nRT(2\ln 2 - 1)$



Question: A protein & Li^{3+} are accelerated through same potential. Difference. If $M_{Li}^{3+} = 8.3 M_p$, then find ratio of de-Broglie wavelength of $\frac{\lambda Li^{3+}}{\lambda_p}$?

Options:
(a) 0.1
(b) 0.2
(c) 0.3
(d) 0.4
Answer: (b)
Solution:

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqv}}$$

$$\frac{\lambda_{li^{+3}}}{\lambda_p} = \frac{\sqrt{m_p e}}{8.3 m_p \times 3e} = \frac{1}{\sqrt{8.3 \times 3}}$$

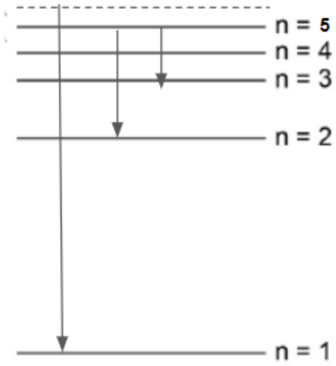
Question: For a transistor emitter current is 40 mA and collector current is 35 mA, then the β of the transistor is?
Options:
(a) $\frac{8}{7}$
(b) $\frac{1}{7}$
(c) $\frac{7}{1}$
(d) $\frac{1}{8}$
Answer: (c)
Solution:

$$\beta = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} = \frac{35}{40-35} = \frac{35}{5} = 7$$

Marks: 4
negativemarks: 1
Subject: Physics
Type: SCQ
Topics:
Targetgrade: JEE_Main-12

Difficulty: Moderate
Analysis:

Question: Choose the correct option (s) regarding the Hydrogen spectrum.



Options:
(a) A is the series limit of Lyman
(b) B is the third line of Balmer
(c) C is the second line of Paschen
(d) All of the above are correct
Answer: (d)
Solution:
A $\rightarrow$ Series limit of laymen
B $\rightarrow$ 3rd line of Balmer
C $\rightarrow$ 2rd line of paschan
All of the options are correct.
Marks: 4
negativemarks: 1
Subject: Physics
Type: SCQ
Topics:
Targetgrade: JEE_Main-12
Difficulty: Moderate
Analysis:

Question: If W = work done, T = Temperature, K_B = Boltzmann constant, x = Displacement & $W = \propto \beta^2 e^{-\frac{x^2}{\propto K_B T}}$, then find dimensions of β?
Options:
(a) $\left[M^1 L^2 T^{-2}\right]$
(b) $\left[M^{-1} L^2 T^{-2}\right]$
(c) $\left[M^1 L^1 T^{-2}\right]$
(d) None of these
Answer: (c)
Solution:
$\frac{x^2}{\alpha k_B T}$ = Dimensionless

$$\alpha = \frac{x^2}{k_B T} = \frac{x^2}{E} \Rightarrow \frac{M^2}{J}$$

$$W = \alpha\beta^2$$

$$\beta^2 = \frac{W}{\alpha} \Rightarrow \frac{J}{\left(M^2 / J\right)} = \frac{J^2}{M^2}$$

$$\beta = \frac{J}{M}$$

Marks: 4
negativemarks: 1
Subject: Physics
Type: SCQ
Topics:
Targetgrade: JEE_Main-12
Difficulty: Moderate
Analysis:

Question: Find the current at time t = 3 seconds, If the charge as a function of time is given as $q(t) = 3t^2 + 2t + 1$.
Options:
(a) 34 A
(b) 21 A
(c) 20 A
(d) 15 A
Answer: (c)
Solution:

$$q = 3t^2 + 2t + 1$$

$$I = \frac{dq}{dt} = 6t + 2$$

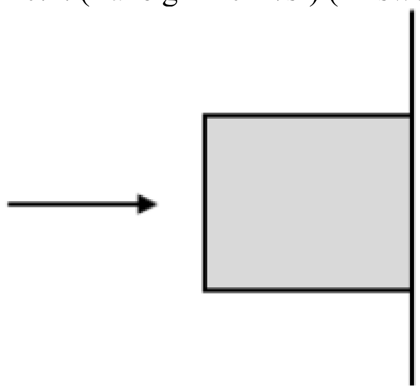
$$I_{t=3} = 6 \times 3 + 2 = 20A$$

Question: Two satellites are revolving in the orbit around a planet. The ratio of their time periods is 1 : 8. Find the ratio of their angular velocities.
Options:
(a) $\frac{w_1}{w_2} = \frac{1}{2}$
(b) $\frac{w_1}{w_2} = \frac{8}{1}$
(c) $\frac{w_1}{w_2} = \frac{2}{1}$
(d) $\frac{w_1}{w_2} = \frac{2}{3}$
Answer: (b)
Solution:

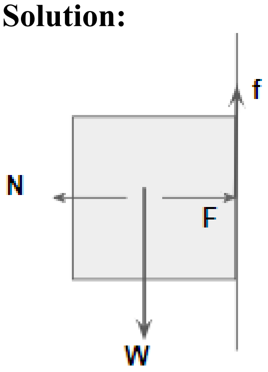
$$\omega = \frac{2\pi}{T}$$

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} = \frac{8}{1}$$
Marks: 4
negativemarks: 1
Subject: Physics
Type: SCQ
Topics:
Targetgrade: JEE_Main-12
Difficulty: Moderate
Analysis:

Question: Find the minimum force F_o that should be applied to the block of mass m = 0.5 kg, so that the block stays in equilibrium with the rough vertical wall having friction coefficient μ = 0.2. (Take g = 10 m/s^2) (Answer to the nearest integer).



Answer: 25.00
Solution:



$N = F$
$f = \mu N = \mu F$
For equilibrium
$W = f = \mu F$
$mg = \mu F$
$$F = \frac{mg}{\mu} = \frac{0.5 \times 10}{0.2} = 25\,N$$

Question: A 220 volts AC supply is given to the primary circuit of transformer and as output of 12 volts DC is taken out using rectifier. If secondary number of turns was 24, then find the no. of turns in primary coil.
Answer: (440)

Solution:

$$\frac{N_P}{N_S}=\frac{V_P}{V_S}$$

$$\frac{N_P}{24}=\frac{220}{12};\quad N_P=440$$

Question: A vertical cross-section of plane is $y=\frac{x^2}{4}$, coefficient of friction is 0.5. Find maximum height at which particle can stay (in cm).

Answer: (25)

Solution:

$$\mu \ge \tan\theta=\frac{dy}{dx}=\frac{dx}{4}=\frac{x}{2}$$

$$0.5\ge\frac{x}{2}$$

$$x\le 1$$

$$\sqrt{4y}\le 1$$

$$2\sqrt{y}\le 1$$

$$y\le\frac{1}{4}m$$

Question: Two identical particles of same mass are shown just before collision and just after collision than $\frac{v_1}{v_2}$ is

Answer: (1)

Solution:

Using linear momentum conservation in y-direction

$$P_i=0$$

$$P_f=m\times\frac{1}{2}v_1-m\times\frac{1}{2}v_2$$

$$v_1=v_2$$

Question: $i=20t+8t^2$

Find charge flown during $0\le t\le 15$

Answer: (11250)

Solution:

$$\frac{dq}{dt} = \left(20t + 8t^2\right)$$

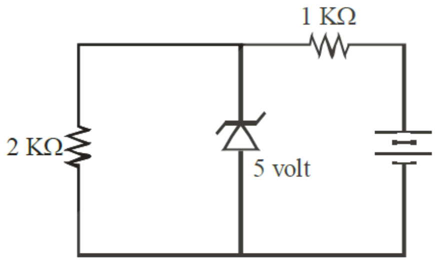
$$\int dq = \int_0^{15} \left(20t + 8t^2\right) dt$$

$$\Delta q = \left[20\frac{t^2}{2} + \frac{8t^3}{3}\right]_0^{15}$$

$$= \frac{20 \times (15)^2}{2} + \frac{8 \times (15)^3}{3}$$

$$\Delta q = 11250\, C$$

Question: A Zener diode is connected in the circuit as shown. The current in the $2k\Omega$ resistance is $x \times 10^{-4}$ A, where x is:.



Answer: (25)
Solution:

$$i = \frac{5}{2 \times 10^{-3}} = 2.5 \times 10^{-3} A$$

Question: A 100 kg block can be lift by placing a mass m on a piston (hydraulic lift). If diameter of piston placed at one and is increased by 4 times and that of second piston is decreased by 4 times. Now same mass m is placed at piston than how many kg weight can be lift up for the new set up?
Answer: (25600)
Solution:

Initially $\frac{100g}{A_1} = \frac{mg}{A_2}$ ...(i)

Initially $\frac{Mg}{16A_1} = \frac{mg}{\left(\frac{A_2}{16}\right)}$ ...(ii)

$$\frac{100 \times 16}{M} = \frac{1}{16} = M = 25600\, kg$$

Question: An electromagnetic wave is propagating in medium, where $\mu_r = \varepsilon_r = 2$. If speed of light in medium is $x \times 10^7\, m/s$. Find out x.
Answer: (15)
Solution:

$$n=\sqrt{\mu_r\varepsilon_r}=2$$

$$v=\frac{C}{n}=\frac{3\times10^8}{2}=15\times10^7\ m/s$$

$$x=15$$

Question: $V_m=20\sin\left[100\pi t+\frac{\pi}{4}\right]$

$$V_c=80\sin\left[10^4\ \pi t+\frac{\pi}{6}\right]$$

For amplitude modulation wave find out percentage modulation index.

Answer: (25)

Solution:

$$m\%=\frac{A_m}{A_c}\times100=\frac{20}{80}\times100=25\%$$

Question: In AC series R-L-C resonance circuit Resistance R = 6.28 ohm, frequency 10 MH Z and self inductance $L=2\times10^{-4}$ Henry is given. Find Quality factor of circuit.

Answer: (2000)

Solution:

$$Q=\frac{x_L}{R}=\frac{\omega L}{R}=\frac{2\pi fL}{R}$$

$$Q=\frac{2\pi\times10^6\times10\times2\times10^{-4}}{6.28}=2000$$

$$Q=2000$$

JEE-Main-24-02-2021-Shift-1

CHEMISTRY

Question: Coordination number of atoms in BCC ?
Options:
(a) 8
(b) 6
(c) 12
(d) 4
Answer: (a)
Solution: Coordination number of BCC is 8

Question: Composition of gun metal?
Options:
(a) 88 % Cu, 10 % Sn, 2 % Zn
(b) 80 % Cu, 10 % Sn, 10 % Zn
(c) 85 % Cu, 10 % Sn, 5 % Zn
(d) 90 % Cu, 8 % Sn, 2 % Zn
Answer: (a)
Solution: Composition of gun metal is
88 % Cu, 10 % Sn, 2 % Zn

Question: Anaerobic respiration causes:
Options:
(a) Global warming
(b) Acid rain
(c) Greenhouse effect
(d) None of these
Answer: (c)
Solution: The by-product of anaerobic respiration is CH_4, which is a Greenhouse gas

Question: Monomer of Buna-S and nylon-6, respectively are
(a) Styrene & caprolactam
(b) Cyclohexane & caprolactam
(c) Ethylene glycol & styrene
(d) Styrene & amino acid
Answer: (a)
Solution: Monomer of Buna-S is styrene and monomer of nylon-6 is caprolactam

Question: Why nitration of aniline gives 47 % meta product?
Options:
(a) Due to the formation of anilinium ion.
(b) Aniline is ortho/para directing
(c) In acidic medium, aniline is converted into anilinium ion which is ortho/para directing.
(d) None of these.
Answer: (a)

Solution: The anilinium group, no longer possessing a free electron pair, deactivates the aromatic ring towards electrophillic substitution reaction, also anilinium ion which gives meta directive. Hence nitration of aniline gives meta product.

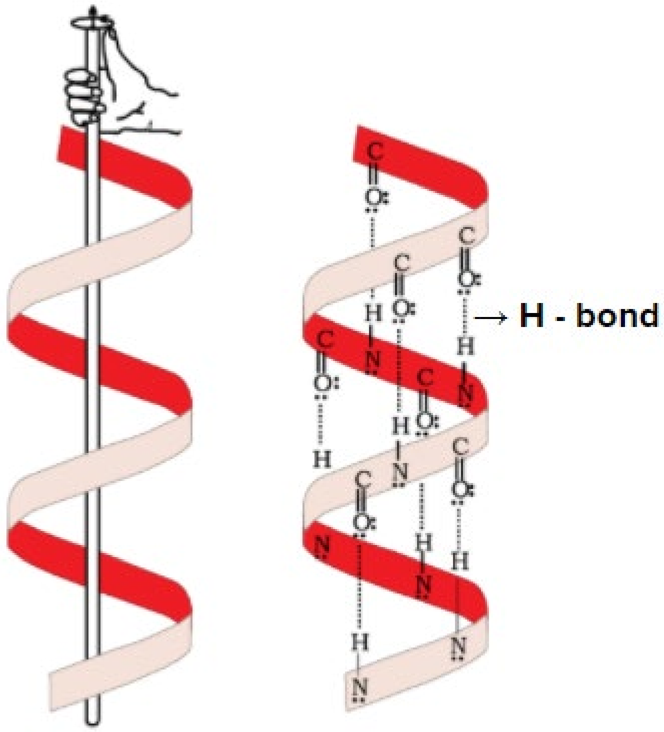
Question: Which of the following ore is concentrated using group 1 cyanide?
Options:
(a) Sphalerite
(b) Malachite
(c) Calamine
(d) Siderite
Answer: (a)
Solution:
Sphalerite → ZnS
Malachite → $CuCO_3.Cu(OH)_2$
Calamine → $CuCO_3$
Siderite → $FeCO_3$
Now, as we know
$ZnS + NaCN \rightarrow [Zn(CN)_4]^{2-}$
Hence, option (a) is correct

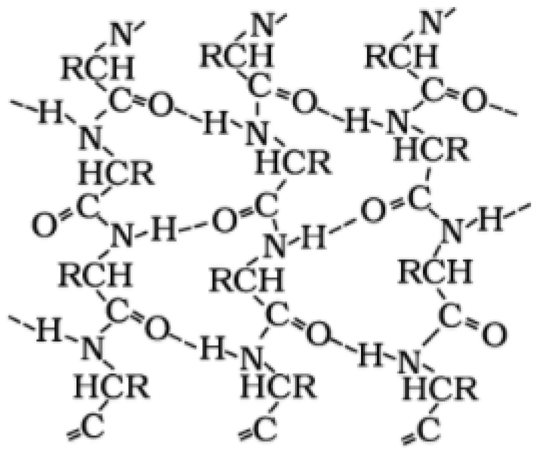
Question: Compare the first Ionisation Energy of Mg, Al, Si, P, S.
Options:
(a) P > S > Si > Mg > Al
(b) P < S < Si < Mg < Al
(c) P > S > Si > Al > Mg
(d) S > P > Si > Al > Mg
Answer: (a)
Solution: I. E $\propto$ fully filled / half-filled configuration
Mg and P have fully filled and half-filled configuration respectively and I.E increasing along the period

Question: An ore of tin containing $FeCrO_4$ is concentrated by :
Options:
(a) Magnetic separation
(b) Electrolytic separation
(c) Hydraulic separation
(d) None of these
Answer: (a)
Solution: Because $FeCrO_4$ is ferromagnetic in nature.

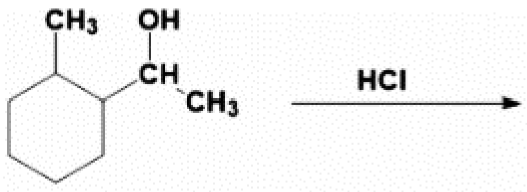
Question: The non-existence of PbI_4 is due to its:
Options:
(a) Small size of Pb^{+4} and large size of I^- ion
(b) High oxidising power of Pb^{+4} ion
(c) High reducing power of I^- ion
(d) Both (b) and (c)
Answer: (d)
Solution: PbI_4 does not exist because pb^{+4} ion has high oxidizing nature & gets immediately reduced to Pb^{+2}. I^- has strong reducing power in PbI_4

Question: Helical structure of protein is formed by which bond?
Options:
(a) H-bond
(b) Peptide bond
(c) Dipeptide bond
(d) None of these
Answer: (a)
Solution:



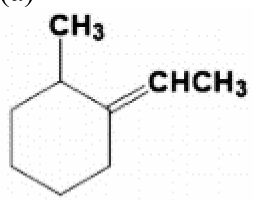


Question:

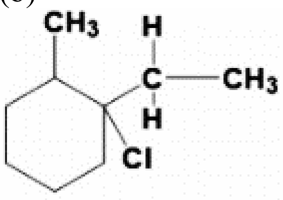


Options:

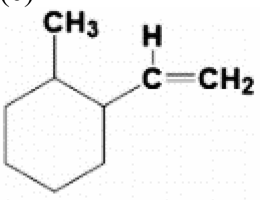
(a)



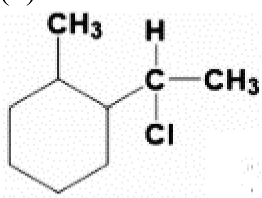
(b)



(c)

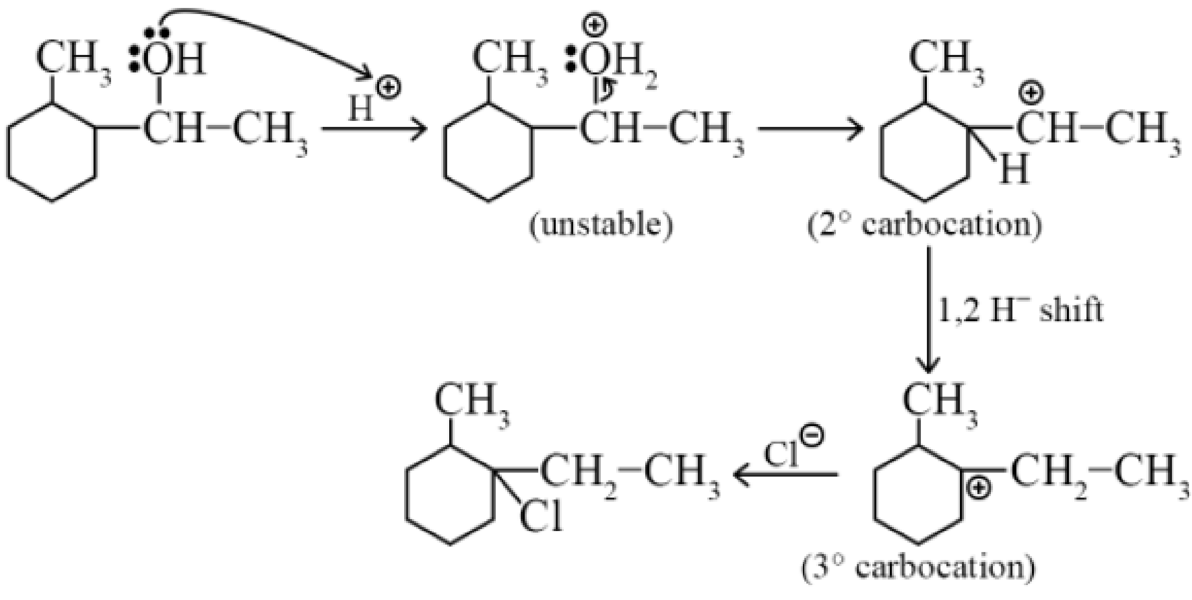


(d)



Answer: (b)

Solution:



Question: Cu, Zn, Cr, Co

Which have +ve reduction potential values for $M^{2+}_{(aq)} / M_{(s)}$

Options:

(a) Cu^{+2}/Cu

(b) Zn^{+2}/Zn

(c) Cr^{+2}/Cr

(d) Co^{+2}/Co

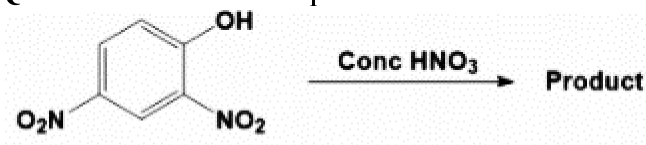
Answer: Cu

Solution:

$$E_{RP} = \frac{-nF}{\Delta G}$$

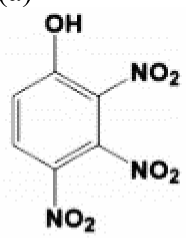
ΔG being –ve for Cu^{+2}/Cu, E_{RP} comes out to be a positive value.

Question: Predominant product is

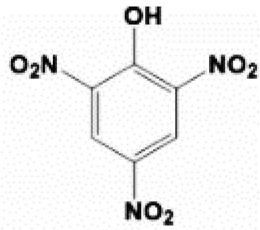


Options:

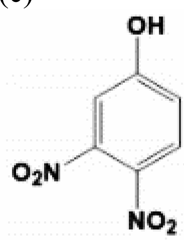
(a)



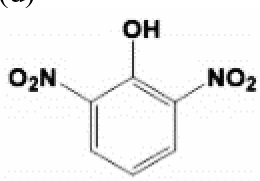
(b)



(c)

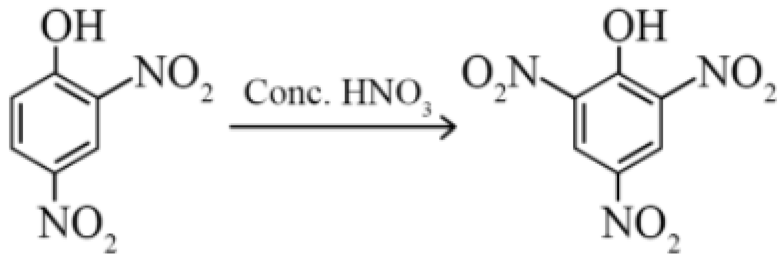


(d)



Answer: (b)

Solution:

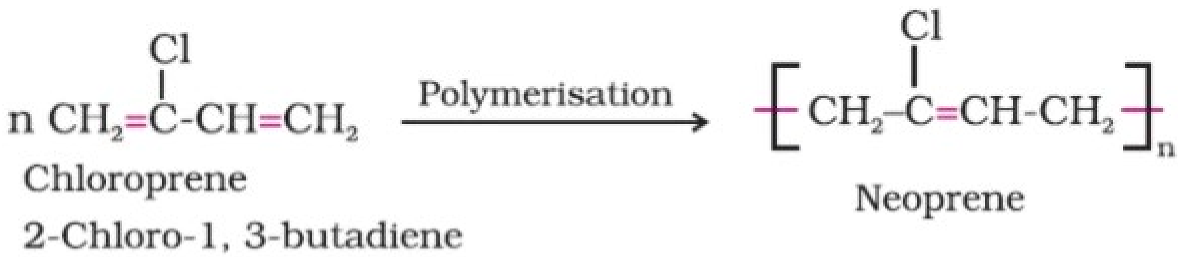


As –OH group is activating in nature

$\therefore$ $\overset{\oplus}{N}O_2$ attacks at ortho and para position, but para position is already blocked

$\therefore$ $\overset{\oplus}{N}O_2$ is only attached at ortho position w.r.t. OH group

Question: Number of chiral carbons in neoprene?
Options:
(a) 0
(b) 1
(c) 2
(d) 3
Answer: (a)
Solution:

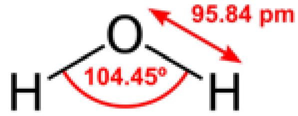


In neoprene no chiral carbon is present

Question: Rate of diffusion does not depend on:
Options:
(a) Size of molecule
(b) Concentration
(c) Temperature
(d) Mass of particle
Answer: (a)
Solution: Rate of diffusion depends on the following factor:
a) Concentration
b) Temperature
c) mass of particle

Question: Bond angle in H_2O?
Options:
(a) 104.5°
(b) 107.5°
(c) 109.28°
(d) None of these

Answer: (a)
Solution: Due to l.p – l.p repulsion of oxygen atom
The bond angle in H2O molecule decrease and becomes 104.5°



Question: Which of the following oxides of nitrogen converts to another oxide on heating?
Options:
(a) NO
(b) N_2O
(c) N_2O_4
(d) All of these
Answer: (c)
Solution:
$N_2O_4 \xrightarrow{\Delta} 2NO_2$

Question: Find the time period of reaction for 40 % completion of reaction.
Rate constant is $25 \times 10^{-4}\ s^{-1}$
Answer: 202.60

$$t = \frac{2.303}{K}\log\left(\frac{100}{100-x}\right)$$
$$= \frac{2.303}{25\times10^{-4}}\times\log\left(\frac{100}{60}\right)$$
$$= \frac{2.303}{25\times10^{-4}}\times0.22$$
$$= 0.02026 \times 10^{-4}$$
$$\Rightarrow 202.60$$

Question: Substance given with molar mass = 90 g per mole.
The weight given is: 4.5 g.
Volume of solution = 250 ml.
Find molarity?
Answer: 0.20
Solution:

$$\text{M} = \frac{\text{moles of solute}}{\text{Volume}(\text{L})}$$
$$= \frac{45}{90}\times\frac{1000}{250\times10} = \frac{1}{5} = 0.2$$

JEE-Main-24-02-2021-Shift-1

MATHEMATICS

Question: The area bounded by region inside the circle $x^2 + y^2 = 36$ and outside the parabola $y^2 = 9x$ is

Options:

(a) $12\pi + 3\sqrt{3}$

(b) $36\pi + 3\sqrt{3}$

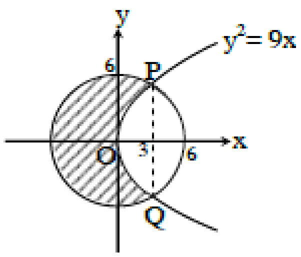
(c) $24\pi + 3\sqrt{3}$

(d) $24\pi - 3\sqrt{3}$

Answer: (c)

Solution:

The curves intersect at points $\left(3, \pm 3\sqrt{3}\right)$



Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36 - x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3}\left|\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right|_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9\pi - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right)$$

$$= 24\pi - 3\sqrt{3}$$

Question: The locus of mid – point of the line segment joining focus of parabola $y^2 = 4ax$ to a point moving on it, is a parabola equation of whose directix is

Options:

(a) $y = 0$

(b) $x = 0$

(c) $x = a$

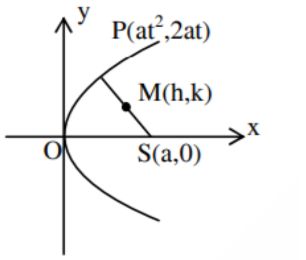
(d) $y = a$

Answer: (b)

Solution:

$h=\frac{at^2+a}{2}, k=\frac{2at+0}{2}$

$\Rightarrow t^2=\frac{2h-a}{a}$ and $t=\frac{k}{a}$



$\Rightarrow \frac{k^2}{a^2}=\frac{2h-a}{a}$

$\Rightarrow$ Locus of (h,k) is $y^2=a(2x-a)$

$\Rightarrow y^2=2a\left(x-\frac{a}{2}\right)$

Its directix is $x-\frac{a}{2}=-\frac{a}{2}\Rightarrow x=0$

Question: The equation of plane perpendicular to plans $3x+y-2z+1=0$ and $2x-5y-z+3=0$ such that it passes through point (1, 2, -3)

Options:

(a) $11x+y+17z+38=0$

(b) $11x-y-17z+40=0$

(c) $11x+y-17z+36=0$

(d) $x+11y+17z+3=0$

Answer: (a)

Solution:

Normal vector of required plane is $\overline{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -2\\ 2 & -5 & -1\end{vmatrix}=-11\hat{i}-\hat{j}-17\hat{k}$

$\therefore 11(x-1)+(y-2)+17(z+3)=0$

$11x+y+17z+38=0$

Question: There are 6 Indians 8 foreigners

Find number of committee form with atteast 2 Indians such numbers of foreigners is twice the number of Indians.

Options:

(a) 1625

(b) 1050

(c) 1400

(d) 575

Answer: (a)

Solution:

$(2I,4F)+(3I,6F)+(4I,8F)$

$= {}^6C_2\,{}^8C_4 + {}^6C_3\,{}^8C_6 + {}^6C_4\,{}^8C_8$

$=15\times 70+20\times 28+15\times 1$

$=1050+560+15=1625$

Question: If $f: R \to R$ is a function defined by $f(x)=[x-1]\cos\left(\frac{2x-1}{2}\right)\pi,$ where [*x*] denotes the greatest integer function, then *f* is

Options:

(a) continuous for every real *x*.

(b) discontinuous only at *x* = 1.

(c) discontinuous only at non-zero integral values of *x*

(d) continuous only at *x* = 1.

Answer: (a)

Solution:

Doubtful points are $x=n,\ n\in I$

$L.H.L=\lim_{x\to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi=(n-2)\cos\left(\frac{2n-1}{2}\right)\pi=0$

$R.H.L=\lim_{x\to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi=(n-2)\cos\left(\frac{2n-1}{2}\right)\pi=0$

$f(n)=0$

Hence continuous

Question: There are two positive number p and q such that $p+q=2$ and $p^4+q^4=272$. Find the quadratic equation whose roots are p and q

Options:

(a) $x^2-2x+2=0$

(b) $x^2-2x+135=0$

(c) $x^2-2x+16=0$

(d) $x^2-2x+130=0$

Answer: (c)

Solution:

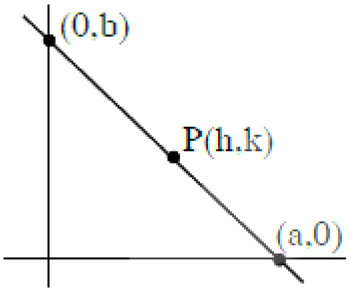
$$\left(p^2+q^2\right)^2-2p^2q^2=272$$
$$\left((p+q)^2+2pq\right)^2-2p^2q^2=272$$
$$16-16pq+2p^2q^2=272$$
$$(pq)^2-8pq-128=0$$
$$pq=\frac{8\pm 24}{2}=16,-8$$
$$pq=16$$

Question: A point is moving on the line such that the AM of reciprocal of intercepts on axis is $\frac{1}{4}$. There are 3 stones whose position are (2, 2) (4, 4) and (1, 1). Find the stone which satisfies the line

Options:
(a) (2, 4)
(b) (4, 4)
(c) (1, 1)
(d) none of these

Answer: (a)

Solution:



$$\frac{x}{a}+\frac{y}{b}=1$$
$$\frac{h}{a}+\frac{k}{b}=1 \quad ...(i)$$
$$\frac{\frac{1}{a}+\frac{1}{b}}{2}=\frac{1}{4}$$
$$\therefore \frac{1}{a}+\frac{1}{b}=\frac{1}{2} \quad ...(ii)$$
$\therefore$ Lines passes through fixed point (2, 2)
(from (1) and (2))

Question: A fair die is thrown n times. The probability of getting an odd number twice is equal to that getting an even number thrice. The probability of getting an odd number, odd number of times is

Options:

(a) $\frac{1}{3}$

(b) $\frac{1}{6}$

(c) $\frac{1}{2}$

(d) $\frac{1}{8}$

Answer: (c)

Solution: P(odd no. twice) = P(even no. thrice)

$\Rightarrow {}^{n}C_2\left(\frac{1}{2}\right)^n = {}^{n}C_3\left(\frac{1}{2}\right)^n \Rightarrow n = 5$

Success is getting an odd number then P(Odd successes) $= P(1) + P(3) + P(5)$

$= {}^{5}C_1\left(\frac{1}{2}\right)^5 + {}^{5}C_3\left(\frac{1}{2}\right)^5 + {}^{5}C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{1}{2}$

Question: If $e^{\left(\cos^2\theta+\cos^4\theta+\ldots\infty\right)\ln 2}$ is a root of equation $t^2 - 9t + 8$ then the value of $\frac{2\sin\theta}{\sin\theta+\sqrt{3}\cos\theta}$ when $0 < \theta < \frac{\pi}{2}$, is

Options:

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 4

Answer: (a)

Solution:

$e^{\left(\cos^2\theta+\cos^4\theta+\ldots\infty\right)\ln 2} = 2^{\cos^2\theta+\cos^4\theta+\ldots\infty}$

$= 2\cot^2\theta$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2\theta} = 1, 8 \Rightarrow \cot^2\theta = 0, 3$

$0 < \theta < \frac{\pi}{2} \Rightarrow \cot\theta = \sqrt{3}$

$\Rightarrow \frac{2\sin\theta}{\sin\theta+\sqrt{3}\sin\theta} = \frac{2}{1+\sqrt{3}\cos\theta} = \frac{2}{4} = \frac{1}{4}$

Question: Population of a town at time t is given by the differential equation $\frac{dP(t)}{dt} = (0.5)P(t) - 450$. Also $P(0) = 850$ find the time when population of town becomes zero.

Options:

(a) $\ell n9$

(b) $3\ell n4$

(c) $2\ell n18$

(d) $\ell n18$

Answer: (c)

Solution:

$$\frac{dP(t)}{dt} = \frac{P(t) - 900}{2}$$

$$\int_0^t \frac{dP(t)}{P(t) - 900} = \int_0^t \frac{dt}{2}$$

$$\left\{\ell n\left|P(t) - 900\right|\right\}_0^t = \left\{\frac{t}{2}\right\}_0^t$$

$$\ell n\left|P(t) - 900\right| - \ell n\left|P(0) - 900\right| = \frac{t}{2}$$

$$\ell n\left|P(t) - 900\right| - \ell n50 = \frac{t}{2}$$

Let at $t = t_1, P(t) = 0$ hence

$$\ell n\left|P(t) - 900\right| - \ell n50 = \frac{t_1}{2}$$

$$t_1 = 2\ell n18$$

Question: If $I = \int \frac{\cos\theta - \sin\theta}{\sqrt{8 - \sin 2\theta}} d\theta = a\sin^{-1}\left(\frac{\sin\theta + \cos\theta}{b}\right) + C$ then ordered pair is (a, b) is

Options:

(a) (1, 3)

(b) (3, 1)

(c) (1, 1)

(d) (-1, 3)

Answer: (a)

Solution:

Put $\sin\theta + \cos\theta = t \Rightarrow 1 + \sin 2\theta = t^2$

$\Rightarrow (\cos\theta - \sin\theta)d\theta = dt$

$$\therefore\ I = \int \frac{dt}{\sqrt{8 - (t^2 - 1)}} = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + C = \sin^{-1}\left(\frac{\sin\theta + \cos\theta}{3}\right) + C$$

$\Rightarrow a = 1$ and $b = 3$

Question: Which of following is tautology ?

Options:

(a) $A \wedge (A \to B) \to B$

(b) $B \to (A \wedge A \to B)$

(c) $A \wedge (A \vee B)$

(d) $(A \vee B) \wedge A$

Answer: (a)

Solution:

$A \wedge (\sim A \vee B) \to B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \to B$

$= (A \wedge B) \to B$

$= \sim A \vee \sim B \vee B$

$= t$

Question: Such that $f : R \to R,\ f(x) = 2x - 1,\ g(x) = \dfrac{x - \frac{1}{2}}{x - 1}$, then $f(g(x))$ is

Options:

(a) one-one, onto

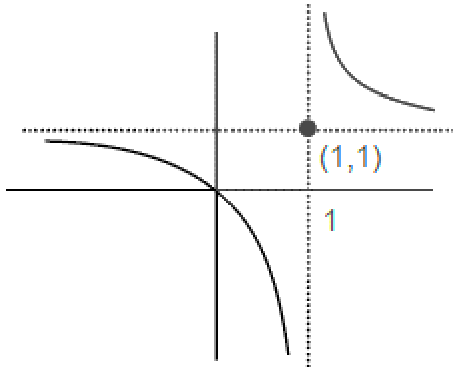
(b) many-one, onto

(c) one-one, into

(d) many-one, into

Answer: (c)

Solution:



$f(g(x)) = 2g(x) - 1$

$= 2\dfrac{\left(x - \frac{1}{2}\right)}{x - 1} = \dfrac{x}{x - 1}$

$f(g(x)) = 1 + \dfrac{1}{x - 1}$

One-one, into

Question: The value of $\left(-{}^{15}C_1+2.{}^{15}C_2-3{}^{15}C_3+.....-15\,{}^{15}C_{15}\right)+\left({}^{14}C_1+{}^{14}C_3+.....+{}^{14}C_{11}\right)$ is

Options:

(a) $2^{16}-14$

(b) $2^{13}-14$

(c) $2^{13}-13$

(d) 2^{14}

Answer: (b)

Solution:

$$S_1=-{}^{15}C_1+2.{}^{15}C_2-.......-15.{}^{15}C_{15}$$

$$=\sum_{r=1}^{15}(-1)^r.r.^{15}=15\sum_{r=1}^{15}(-1)^r\,{}^{14}C_{r-1}$$

$$=15\left(-{}^{14}C_0+{}^{14}C_1-.....-{}^{14}C_{14}\right)=15(0)=0$$

$$S_2={}^{14}C_1+{}^{14}C_3+.....+{}^{14}C_{11}$$

$$=\left({}^{14}C_1+{}^{14}C_3+....+{}^{14}C_{11}+{}^{14}C_{13}\right)-{}^{14}C_{13}$$

$$=2^{13}-14$$

$$S_1+S_2=2^{13}-14$$

Question: The distance of the point P(1, 1, 9) from the point of intersection of plane $x+y+z=17$ and line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$

Options:

(a) $\sqrt{38}$

(b) $\sqrt{39}$

(c) 6

(d) 7

Answer: (a)

Solution:

$$\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda$$

$\Rightarrow x=\lambda+3,\ y=2\lambda+4,\ z=2\lambda+5$

Which lies on given plane hence

$\Rightarrow \lambda+3+2\lambda+4+2\lambda+5=17$

$\Rightarrow \lambda=\frac{5}{5}=1$

Hence, point of intersection is Q (4, 6, 7)

$\therefore$ Required distance = PQ

$$=\sqrt{9+25+4}$$

$$=\sqrt{38}$$

Question: Two towers are 150 m distance apart. Height of one tower is thrice the other tower. The angle of elevation of top of tower from midpoint of their feets are complement to each other height of smaller tower is

Options:

(a) $25\sqrt{3}m$

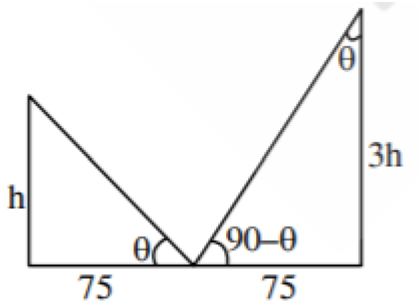
(b) $\frac{25}{\sqrt{3}}m$

(c) $75\sqrt{3}m$

(d) $25m$

Answer: (a)

Solution:



$$\tan\theta = \frac{h}{75} = \frac{75}{3h}$$

$$\Rightarrow h^2 = \frac{(75)^2}{3}$$

$$h = 25\sqrt{3}m$$

Question: The value of $\lim\limits_{x\to 0^+} \frac{\int_0^{x^2} \sin\sqrt{t}\,dt}{x^3}$

Options:

(a) $\frac{1}{15}$

(b) $\frac{2}{3}$

(c) 3

(d) 2

Answer: (b)

Solution:

$$\lim_{x\to 0^+} \frac{\int_0^{x^2} \sin\sqrt{t}\,dt}{x^3} = \lim_{x\to 0^+} \frac{(\sin|x|)2x}{3x^2} = \lim_{x\to 0^+}\left(\frac{\sin x}{x}\right)\times\frac{2}{3} = \frac{2}{3}$$

Question: Tangent at point $P(t,t^3)$ of curve $y=x^3$ meets the curve again at Q then ordinate of point which divides PQ in $1:2$ internally, is

Options:

(a) 0

(b) $2t^3$

(c) $-2t^3$

(d) $8t$

Answer: (c)

Solution: equation of tangent at $P(t,t^3)$

$(y-t^3)=3t^2(x-t)$..........(i)

now solve the above equation with

$y=x^3$..........(ii)

By (i) & (ii)

$x^3-t^3=3t^2(x-t)$

$x^2+xt+t^2=3t^2$

$x^2+xt-2t^2=0$

$(x-t)(x+2t)=0$

$\Rightarrow x=-2t$

$\Rightarrow Q(-2t,-8t^3)$

Ordinate of required point $=\dfrac{2t^3+(-8t^3)}{3}=-2t^3$

Question: The values of k and m such that system of equations

$3x+2y-kz=10,\ x-2y+3z=3,\ x+2y-3z=5m$ are inconsistent.

Options:

(a) $k=3$ and $m\neq\dfrac{7}{3}$

(b) $k=3$ and $m\neq\dfrac{7}{10}$

(c) $k\neq3$ and $m=\dfrac{7}{10}$

(d) $k=2$ and $m\neq\dfrac{7}{10}$

Answer: (a)

Solution:

$$\Delta=\begin{vmatrix}3 & 2 & -k\\ 1 & -2 & 3\\ 1 & 2 & -3\end{vmatrix}=0\Rightarrow k=3$$

$$\Delta_x = \begin{vmatrix} 10 & 2 & -3 \\ 3 & -2 & 3 \\ 5m & 2 & -3 \end{vmatrix} = 0$$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 1 & 3 & 3 \\ 1 & 5m & -3 \end{vmatrix} = 6(7-10m)$$

$$\Delta_z = \begin{vmatrix} 3 & 2 & 10 \\ 1 & -2 & 3 \\ 1 & 2 & 5m \end{vmatrix} = 4(7-10m)$$

Hence, $k=3$ and $m \neq \frac{7}{10}$

Question: Let $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$ then $f(x)$

Options:

(a) decreases in $\left[\frac{1}{2}, \infty\right)$

(b) increase in $\left[\frac{1}{2}, \infty\right)$

(c) decreases in $(-\infty, \infty)$

(d) increase in $\left(-\infty, \frac{1}{2}\right]$

Answer: (b)

Solution: $f'(x) = (2x-1)(x - \sin x)$

$\Rightarrow f'(x) \geq 0$ in $x \in \left[\frac{1}{2}, \infty\right)$

and $f'(x) \leq 0$ in $x \in \left(-\infty, \frac{1}{2}\right]$

Question: The least value of α such that $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $x \in \left(0, \frac{\pi}{2}\right)$

Answer: 9.00

Solution:

Let $f(x) = \frac{4}{\sin x} + \frac{1}{1-\sin x}$

$$y=\frac{4-3\sin x}{\sin x(1-\sin x)}$$

Let $\sin x=t$ when $t\in(0,1)$

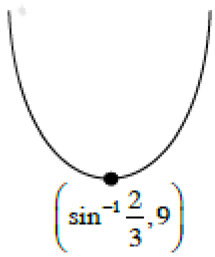
$$y=\frac{4-3t}{t-t^2}$$

$$\frac{dy}{dt}=\frac{-3\left(t-t^2\right)-(1-2t)(4-3t)}{\left(t-t^2\right)^2}=0$$

$\Rightarrow 3t^2-3t-\left(4-11t+6t^2\right)=0$

$\Rightarrow 3t^2-8t+4=0$

$\Rightarrow 3t^2-6t-2t+4=0$



$\Rightarrow t=\frac{2}{3}$

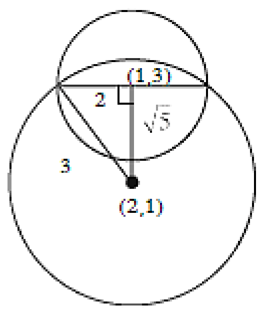
$\Rightarrow \alpha\geq 9$

Least α is equal to 9

Question: One of the diameter of circle $C_1: x^2+y^2-2x-6y+6=0$ is chord of circle C_2 with centre (2, 1) then radius of C_2 is

Answer: 3.00

Solution:



Distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2+(2)^2=r^2$

$\Rightarrow r=3$

Question: $\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r^2+r}\right)\right)=$

Answer: 1.00

Solution:

$$\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$$

$$=\tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$$

$$=\tan\left(\frac{\pi}{4}\right)=1$$

Question: Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in R$. Suppose $Q=\left[q_{ij}\right]$ is a matrix such that $PQ=kI$, where $k\in R,\ k\neq 0$ and I is the identity matrix of order 3. If $q_{23}=-\frac{k}{8}$ and $\det(Q)=\frac{k^2}{2}$, then value of $k^2+\alpha^2$ is equal to

Answer: 17.00

Solution:

As $PQ=kI\Rightarrow Q=kP^{-1}I$

Now $Q=\frac{k}{|P|}(adj\,P)I\Rightarrow Q=\frac{k}{(20+12\alpha)}\begin{bmatrix}- & - & -\\ - & - & (-3\alpha-4)\\ - & - & -\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$\therefore\ q_{23}=\frac{-k}{8}\Rightarrow\frac{k}{(20+12\alpha)}(-3\alpha-4)=\frac{-k}{8}$

$\Rightarrow 2(3\alpha)+4=5+3\alpha$

$3\alpha=-3\Rightarrow\alpha=-1$

Also $|Q|=\frac{k^3|I|}{|P|}\Rightarrow\frac{k^2}{2}=\frac{k^3}{(20+12\alpha)}$

$(20+12\alpha)=2k\Rightarrow 8=2k\Rightarrow k=4$

Question: Of the three independent events B_1, B_2 and B_3, the probability that only B_2 occurs is β and only B_3 occurs is γ. Let the probability p that none of events B_1, B_2 or B_3 occurs satisfy the equations $(\alpha-2\beta)p=\alpha\beta$ and $(\beta-3\gamma)p=2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then $\dfrac{\text{Probability of occurence of } B_1}{\text{Probability of occurence of } B_3} =$

Answer: 6.00

Solution:

Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1-y)(1-z) = \alpha \ \Rightarrow y(1-x)(1-z) = \beta$

$\Rightarrow z(1-x)(1-y) = \gamma \Rightarrow (1-x)(1-y)(1-z) = P$

Putting in the given relation we get $x = 2y$ and $y = 3z \ \Rightarrow x = 6z \ \Rightarrow \ \dfrac{x}{z} = 6$

Question: How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 7?

Answer: 540.00

Solution:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I: Seven (1's) and two (0's)

${}^9C_2 = 36$

Case II: One (2) and three (1's) and five (0's)

$\dfrac{9!}{5!3!} = 504$

$\therefore$ Total = 540

Question: $\vec{c}$ is coplanar with $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = 2\hat{i} + \hat{k}$, $\vec{a}.\vec{c} = 7$ & $\vec{c} \perp \vec{b}$, then the value of $2\left|\vec{a} + \vec{b} + \vec{c}\right|^2$ is

Answer: 75.00

Solution:

$\vec{c} = \lambda\left(\vec{b} \times \left(\vec{a} \times \vec{b}\right)\right)$

$\lambda = \left(\left(\vec{b}.\vec{b}\right)\vec{b} - \left(\vec{b}.\vec{a}\right)\vec{b}\right)$

$= \lambda\left(5\left(-\hat{i} + \hat{j} + \hat{k}\right) + 2\hat{i} + \hat{k}\right)$

$= \lambda\left(-3\hat{i} + 5\hat{j} + 6\hat{k}\right)$

$\vec{c}.\vec{a} = 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$

$\lambda = \dfrac{1}{2}$

$\therefore\ 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2$

$=2\left(\frac{1}{4}+\frac{49}{4}+25\right)=25+50=75$

Question: $z+\alpha|z-1|+2i=0;\ z\in C\ \&\ \alpha\in R,$ then the value of $4\left[(\alpha_{\max})^2+(\alpha_{\min})^2\right]$ is

Answer: 10.00

Solution:

$x+iy+\alpha\sqrt{(x-1)^2+y^2}+2i=0$

$\therefore\ y+2=0$ and $x+\alpha\sqrt{(x-1)^2+y^2}=0$

$y=-2\ \&\ x^2=\alpha^2\left(x^2-2x+1+4\right)$

$\alpha^2=\frac{x^2}{x^2-2x+5}$

$\alpha^2\in\left[0,\frac{5}{4}\right]$

$\therefore\ \alpha^2\in\left[0,\frac{5}{4}\right]$

$\therefore\ \alpha\in\left[-\frac{\sqrt{5}}{2},\frac{\sqrt{5}}{2}\right]$

Then $4\left[(\alpha_{\max})^2+(\alpha_{\min})^2\right]=4\left(\frac{5}{4}+\frac{5}{4}\right)=10$

Question: Let $A=\{x:x \text{ is 3 digit number}\}$

$B=\{x:x=9K+2,\ k\in I\}$

$C=\{x:x=9k+\ell,\ k\in I,\ \ell\in I,\ 0<\ell<9\}$

If sum of elements in $A\cap(B\cap C)$ is 274 × 400 then ℓ is

Answer: 5.00

Solution:

3 digit number of the form $9K+2$ are {101, 109, …., 992}

$\Rightarrow$ Sum equal to $\frac{100}{2}(1093)$

Similarly sum of 3 digit number of the form $9K+5$ is $\frac{100}{2}(1099)$

$\frac{100}{2}(1093)+\frac{100}{2}(1099)=100\times(1096)$

$=400\times 274$

$\Rightarrow \ell=5$

Question: $\int_{-a}^{a} |x|+|x-2| = 22,\ a>2$ then the value of $\int_{-a}^{a} x+[x]$ is

(where [.] represents greatest integer function)

Answer: -3.00

Solution:

$$\int_{-a}^{0}(-2x+2)\,dx+\int_{0}^{2}(x+2-x)\,dx+\int_{2}^{a}(2x-2)\,dx=22$$

$$x^2-2x\Big|_0^{-a}+2x\Big|_0^2+x^2-2x\Big|_2^a=22$$

$$a^2+2a+4+a^2-2a-(4-4)=22$$

$$2a^2=18\Rightarrow a=3$$

$$\int_{-3}^{3}(x+[x])\,dx=-3-2-1+1+2=-3$$